Meson-Baryon Systems and Their Resonances*

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Meson-baryon systems of angular momentum L are analyzed by considering the relevant crossing matrices. The resultant "bootstrap" calculations yield the correct quantum numbers for the observed array of particles and resonances. Specifically it is found that $Y_1^*(1385)$ decays into $(\pi\Lambda)$ but not $(\pi\Sigma)$. Furthermore, a Y_1^* resonance in $(\frac{5}{2}-)$ is predicted which may accommodate the recently discovered $Y_1^*(1765)$.

A SCHEME has recently been proposed by Carruthers,[1] whereby all the nucleon resonances form a family generated in pairs by a single mechanism. We explore here the possibility of doing the same for the Y^* resonances and thus achieve a coherent scheme for the baryons and their resonances. Three results emerge from this treatment: (a) The quantum numbers of the observed experimental resonances appear in a natural way. (b) Two-body decays agreeing with experiment are found. Of special interest is the observation that $Y_1^*(1385) \not\rightarrow \pi+\Sigma$ is a natural consequence of the proposed scheme. (c) The scheme can accomodate yet-to-be-discovered metastable resonant states of prescribed quantum numbers, in particular, the experimentally suggested possible pion-hyperon resonance $Y_1^*(1765)$ [2] with a dominant decay mode into $\pi+\Lambda$.

We consider all baryon resonances consisting of a pseudoscalar meson and a baryon with orbital angular momentum L, $(M,B)_L$ where $L\geq 1$. Thus we are interested in those values of isospin I and total spin J for which a baryon-meson system will be attractive and hence able to form a metastable state. Since we cannot calculate meson-baryon interaction in its complete generality, we make the usual Born approximation [Fig. 1(a)] which is expected to be the dominant contribution—at least for the lower energy baryon resonances. The interaction force of the $(M,B)_L$ system in an (I,J) state will be attractive or repulsive depending on whether the corresponding phase shift is positive or negative. It is found convenient to consider instead of the phase shifts the corresponding partial wave amplitudes f_{IJ} which are related to δ_{IJ} by

$$f_{IJ}=e^{i\delta_{IJ}}\sin\delta_{IJ}/k\,,$$

thus f and δ always have the same sign.

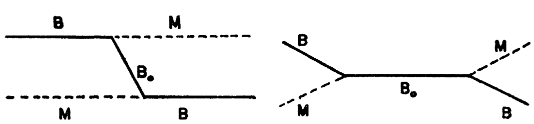

FIG. 1. Crossed and uncrossed diagrams for meson-baryon interaction.

We consider f_{IJ} as a vector indexed by pairs I, J. The crossed process of Fig. 1(b) shown in Fig. 1(a) can be expressed in terms of the uncrossed process as

$$f_{I'J'}{}^c=X_{I'I}{}^{J'J}f_{IJ}\,,$$

where X is the crossing matrix. X is actually the Kronecker product of crossing matrices for I spin and J spin separately

$$X_{I'I}{}^{J'J}=X_{I'I}\otimes X^{J'J}\,.$$

f_{IJ} will be zero for all I, J except that corresponding to the particle exchanged, when it is set equal to unity (no arbitrariness results since we are only interested in the sign and relative magnitude of f^c). The problem is thus reduced to a consideration of relevant crossing matrices.

Mandelstam *et al.*[3] have obtained a general expression for the I-spin crossing matrix between $A+B\rightarrow A+C$ and the crossed reaction $\bar{A}+B\rightarrow\bar{A}+C$

$$X_{I'I}=(-1)^{2a-2I}(2I+1)\begin{Bmatrix} a & b & I \\ a & c & I' \end{Bmatrix}\,.$$

Thus $X_{I'I}$ is proportional to a $6j$ symbol; a, b, c being the I spins of particles A, B, C. Since all baryon-pseudoscalar meson systems are identical with respect to J spin, the same angular-momentum crossing matrix will be used throughout our considerations. We use the expression derived by Carruthers[1] which is evaluated at the meson-baryon threshold and thus represents essentially a static limit

$$\{X^{J'J}\}=\frac{1}{(2L+1)}\begin{Bmatrix} -1 & 2(L+1) \\ 2L & +1 \end{Bmatrix}\,.$$

Note that the relevant $6j$ symbol of $X_{I'I}$ is invariant with respect to the exchange $I\leftrightarrow I'$. The multiplicative factor does change in magnitude, but retains its sign under the transformation. Hence $X_{I'I}\simeq X_{II'}$ with respect to sign. The same is true of $X^{JJ'}$ with respect to $J\leftrightarrow J'$. If exchange of a particle B_0 generates attraction in a state corresponding to B_1 say, then exchange of B_1 must therefore result in an attractive state for B_0. We thus have a bootstrap which may sustain the two states.

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[2] A. Barbaro-Galtieri, A. Hussain, and R. D. Tripp, Phys. Letters **6**, 296 (1963).
[3] S. Mandelstam, J. E. Paton, R. F. Peierls, and A. Q. Sarker, Ann. Phys. (N. Y.) **18**, 198 (1962).

TABLE I. Meson-baryon systems and their crossing matrices.

	System	Crossing Matrix X
(1)	$\pi N \to \pi N$	$\frac{1}{3(2L+1)}\begin{Bmatrix} -1 & +4 \\ +2 & +1 \end{Bmatrix} \times \begin{Bmatrix} -1 & 2(L+1) \\ 2L & +1 \end{Bmatrix}$
(2)	$\pi\Lambda \to \pi\Lambda$	$\frac{1}{(2L+1)}\begin{Bmatrix} -1 & 2(L+1) \\ 2L & +1 \end{Bmatrix}$
(3)	$\pi\Sigma \to \pi\Sigma$	$\frac{1}{3(2L+1)}\begin{Bmatrix} +1 & -3 & +5 \\ -1 & +\frac{3}{2} & +\frac{5}{2} \\ +1 & +\frac{3}{2} & +\frac{1}{2} \end{Bmatrix} \times \begin{Bmatrix} -1 & 2(L+1) \\ 2L & +1 \end{Bmatrix}$
(4)	$\pi\Lambda \to \pi\Sigma$	$\frac{-1}{(2L+1)}\begin{Bmatrix} -1 & 2(L+1) \\ 2L & +1 \end{Bmatrix}$
(5)	$\bar{K}N \to \bar{K}N$	$\frac{1}{2(2L+1)}\begin{Bmatrix} +1 & -1 \\ -3 & -1 \end{Bmatrix} \times \begin{Bmatrix} -1 & 2(L+1) \\ 2L & +1 \end{Bmatrix}$

To consider specific cases, the crossing matrices for the various $(M,B)_L$ systems are written down (Table I). We then find all states allowed by strong interaction conservation laws [I spin, J spin and parity (P)] for the system and therefore allowed for the exchanged particle also. These are tabulated in Table II (first column). If we successively used all these states for the particle B_0, a great many attractive pairs (B_0,B_1) would result, and most of these would be mutually interfering. Unless one such pair is very dominant, no *ad hoc* theory or criterion to eliminate some of the redundant pairs is available. Indeed such a theory might not even be very useful because we are working with the Born approximation only. At higher energies especially, we expect closed channels and multiparticle states to enter into the dynamics significantly so that numerical values of the crossing matrix become less reliable.

To proceed further, we notice that the feasible bootstraps for any L correspond directly to the bootstraps of any other L, because the essential structure of the crossing matrix does not depend on L. Therefore if we decide for one L which of the bootstraps is the actually occurring one, we can assume that for other L it will be precisely the corresponding bootstrap which will be realized, as first suggested by Carruthers.[1] For the lowest angular momentum we shall always take the P wave. It will be seen later that this yields a complete array of particles and resonances.[4] If then we can decide

TABLE II. Allowed states and preferred bootstraps for meson-baryon systems.

	Available states I	J	P	Lowest states L	$L=1$	$XB_0 \to B_1$	$XB_1 \to B_0$
(1)	$\begin{matrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{3}{2} \\ \frac{3}{2} \end{matrix}$	$\begin{matrix} L-\frac{1}{2} \\ L+\frac{1}{2} \\ L-\frac{1}{2} \\ L+\frac{1}{2} \end{matrix}$	$(-1)^{L+1}$	$N_{1/2,\,L-1/2}{}^P$	$N_{1/2,\,1/2}{}^+$	$\frac{1}{3(2L+1)}\begin{bmatrix} +1 \\ -2L \\ -2 \\ +4L \end{bmatrix} \to N_{3/2,\,L+1/2}{}^P$	$\frac{1}{3(2L+1)}\begin{bmatrix} 8(L+1) \\ +4 \\ 2(L+1) \\ +1 \end{bmatrix} \to N_{1/2,\,L-1/2}{}^P$
(2)	$\begin{matrix} 1 \\ 1 \end{matrix}$	$\begin{matrix} L-\frac{1}{2} \\ L+\frac{1}{2} \end{matrix}$	$(-1)^{L+1}$	$\Sigma_{1,\,L-1/2}{}^P$	$\Sigma_{1,\,1/2}{}^+$	$\frac{1}{(2L+1)}\begin{bmatrix} -1 \\ 2L \end{bmatrix} \to \Sigma_{1,\,L+1/2}{}^P$	$\frac{1}{(2L+1)}\begin{bmatrix} 2(L+1) \\ +1 \end{bmatrix} \to \Sigma_{1,\,L-1/2}{}^P$
(3)	$\begin{matrix} 0 \\ 0 \\ 1 \\ 1 \\ 2 \\ 2 \end{matrix}$	$\begin{matrix} L-\frac{1}{2} \\ L+\frac{1}{2} \\ L-\frac{1}{2} \\ L+\frac{1}{2} \\ L-\frac{1}{2} \\ L+\frac{1}{2} \end{matrix}$	$(-1)^{L+1}$	$\Lambda_{0,\,L-1/2}{}^P$	$\Lambda_{0,\,1/2}{}^+$	$\frac{1}{3(2L+1)}\begin{bmatrix} -1 \\ 2L \\ +1 \\ -2L \\ -1 \\ 2L \end{bmatrix} \to \Sigma_{1,\,L-1/2}{}^P$	$\frac{1}{3(2L+1)}\begin{bmatrix} +3 \\ -6L \\ -\frac{3}{2} \\ +3L \\ -\frac{3}{2} \\ +3L \end{bmatrix} \to \Lambda_{0,\,L-1/2}{}^P$
(5)	$\begin{matrix} 0 \\ 0 \\ 1 \\ 1 \end{matrix}$	$\begin{matrix} L-\frac{1}{2} \\ L+\frac{1}{2} \\ L-\frac{1}{2} \\ L+\frac{1}{2} \end{matrix}$	$(-1)^{L+1}$	$\Lambda_{0,\,L-1/2}{}^P$	$\Lambda_{0,\,1/2}{}^+$	$\frac{1}{2(2L+1)}\begin{bmatrix} -1 \\ 2L \\ +3 \\ -6L \end{bmatrix} \to \Sigma_{1,\,L-1/2}{}^P$	$\frac{1}{2(2L+1)}\begin{bmatrix} +1 \\ -2L \\ +1 \\ -2L \end{bmatrix} \to \Lambda_{0,\,L-1/2}{}^P$

[4] For the π-N problem the consistency of neglecting $L=0$ has been analyzed by D. Amati and S. Fubini, Ann. Rev. Nucl. Sci. **12**, 359 (1962).

which bootstrap actually occurs for $L=1$, we shall have a whole family of bootstraps for successive angular momenta L.

In order to decide on the preferred bootstrap, we shall look for the stable baryons among the P states and select that bootstrap containing one of them. If this selection procedure yields more than one feasible bootstrap, we require further that the bootstrap connecting two stable baryons be chosen. Naturally the resultant bootstrap must be the actually occurring one, since it yields the observed particles which, of course, we have already assumed in the first place, when setting up the various systems. The assumptions here employed, generalized to include the hyperons, are actually not much more extensive than those used when treating the pion-nucleon situation. They do lead to a unique resonant structure for the meson-baryon systems in agreement with experimental evidence.

In general the stable baryons will be contained as the lowest member in L of the series with lowest quantum numbers (I,J), which we shall therefore use as our B_0 (Table II, second column). Applying the crossing matrix to B_0 yields both attractive and repulsive states. If only one of them is attractive, it is naturally the desired B_1 [as for systems (1) and (2) of Table II]. Otherwise we must select for B_1 that series in L which contains a stable baryon for $L=1$ [as in systems (3) and (5)]. Applying X to the chosen B_1 we shall get back B_0. A number of families result and they are listed in Table II [(1) to (5)] with the appropriate systems and crossing matrices given in Table I. We comment on these briefly.

(1) $N_{1/2,L+1/2}{}^P \leftrightarrow N_{3/2,L+1/2}{}^P$, where P denotes the parity. This is the Carruthers proposal. The first pair ($L=1$) forms the bootstrap $N \leftrightarrow N^*$ originally suggested by Chew.[5] Abers and Zemach[6] have made a self-consistent calculation of the nucleon and N^* masses based on this approach. Their results agree reasonably well with the experimental masses and thus give us hope that the static crossing matrices may already provide an essentially correct description of meson-baryon systems. This expectation is reinforced by Carruthers' detailed calculations[1] of higher order contributions (such as recoil) to the crossing matrix, which serve only to accentuate the feasibility of the crossing approach.

(2) $\Sigma_{1,L-1/2}{}^P \leftrightarrow \Sigma_{1,L+1/2}{}^P$. For $L=1$ this forms the $\Sigma \leftrightarrow Y_1^*(1385)$ bootstrap.

(3) $\Lambda_{0,L-1/2}{}^P \leftrightarrow \Sigma_{1,L-1/2}{}^P$. The series $\Lambda_{0,L-1/2}{}^P$ contains baryon $\Lambda_{0,1/2}{}^+$ as the lowest member and is thus taken for B_0. This choice of B_0 generates attractive states for three series of states (in L) simultaneously: $\Lambda_{0,L+1/2}{}^P$, $\Sigma_{1,L-1/2}{}^P$ and $\Sigma_{2,L+1/2}{}^P$. Of these only $\Sigma_{1,L-1/2}{}^P$ contains a stable baryon $\Sigma_{1,1/2}{}^+$ and must therefore be chosen for the bootstrap.[7] If we had taken $\Sigma_{1,L-1/2}{}^P$ for B_0, we would have obtained attraction for $\Lambda_{0,L-1/2}{}^P$, $\Sigma_{1,L+1/2}{}^P$, and $\Sigma_{2,L+1/2}{}^P$ and would be required to choose the $\Lambda_{0,L-1/2}{}^P$ series, yielding, of course, the same bootstrap. Note that this results in Λ being a bound P-wave state of $(\pi,\Sigma)_1$ with Σ exchange, while Σ is yielded by the same system under Λ exchange.[8]

(4) When a resonance is established as an attractive state of an $(M,B)_L$ system, then production as well as decay can naturally be realized *via* this system. To make a statement, however, about which decays are *not* allowed, all the possible Born diagrams have to be examined. For $\pi\Lambda \to \pi\Sigma$, X has no positive off-diagonal elements and hence sustains no bootstrap.

(5) The reasons for selecting states here are the same as in (3), and, in fact, we obtain the same family.

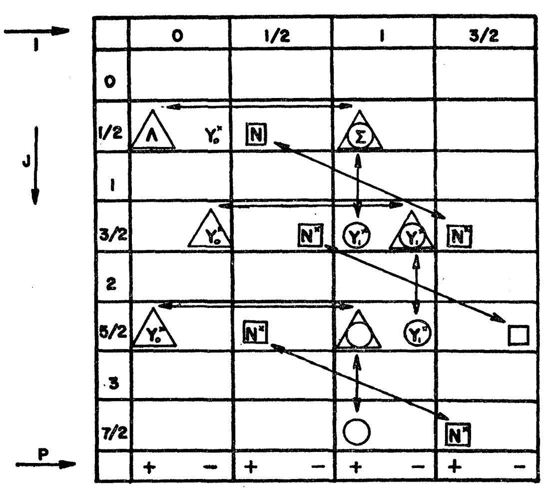

FIG. 2. Resonance families generated by the first three bootstraps (P, D, and F wave). △ denotes $(\bar{K},N)_L$ and $(\pi,\Sigma)_L$ decay, □ is for $(\pi,N)_L$ and ○ for $(\pi,\Lambda)_L$ decay.

The system $\eta N \to \eta N$, $\eta\Lambda \to \eta\Lambda$, $\eta\Sigma \to \eta\Sigma$ have also been considered. For all three cases X is identical to the crossing matrix for $\pi\Lambda \to \pi\Lambda$. Thus bootstraps may be possible between $N_{1/2,L-1/2}{}^P \leftrightarrow N_{1/2,L+1/2}{}^P$, $\Lambda_{0,L-1/2}{}^P \leftrightarrow \Lambda_{0,L+1/2}{}^P$, and $\Sigma_{1,L-1/2}{}^P \leftrightarrow \Sigma_{1,L+1/2}{}^P$. The η decays electromagnetically into three pions, and η-baryon thresholds are fairly high. Since the first two bootstraps are not supported by any other meson-baryon pairs, the corresponding resonances are perhaps difficult to realize in nature. The remaining bootstrap is also yielded by $\pi\Lambda \to \pi\Lambda$ and thus might give support to an $\eta-\Sigma$ bootstrap. The $\eta\Sigma$ threshold lies at 1738 MeV, thus only $Y_1^*(1765)$ appears as a possible candidate. However, since both $\pi\Lambda$ and $\eta\Sigma$ must in this case be in D states, it

[5] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).

[6] E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963). See also B. Diu and H. R. Rubinstein (unpublished).

[7] It can readily be seen that abandonment of our approach will yield arrays of resonant states inconsistent with experimental data (cf. Ref. 9).

[8] In general the $(\pi-\Lambda)$ bootstraps seem to be more important than those of $(\pi-\Sigma)$. We are, of course concerned here with outlining the general system of bootstraps, without detailed analysis of individual cases.

is rather doubtful from phase-space considerations whether the $\eta\Sigma$ decay can really be competitive.

In Fig. 2 we have written down the first three pairs of all the obtained families in an I–J diagram. Note that the system of possible metastable states thus obtained agrees with the actually observed baryon resonances[9] and their quantum numbers (where known). An exception is the absence of $Y_0^*(1405)$ from our considerations, thus lending support to the theory that it is a $(\bar{K}N)_0$ S-wave bound state[10] and hence outside the domain for which our scheme is applicable. This embodies assertion (a) of the Introduction.

It is evident that not only the quantum numbers but also the decays of the resonances are given by our scheme. For the two-body systems considered here, these again agree with observed decays. Neither $\pi+\Sigma\rightarrow\pi+\Sigma$ nor $\pi+\Sigma\rightarrow\pi+\Lambda$ support a bootstrap containing $Y_{1,3/2}{}^+$ and hence in the framework of the Born approximation for pion-hyperon systems, $Y_1^*(1385)$ has only a $(\pi,\Lambda)_1$ decay [assertion (b)].

Our entire approach makes no explicit mention of baryon and meson masses, in this sense the crossing matrices are purely geometric quantities. It is clear, however, that for higher J-spin metastable states will occur at a correspondingly higher energy. The presence of closed channels and multiparticle states become important at higher energies and hence not all attractive states of the static crossing matrices will actually correspond to observed resonances. However our scheme may hold good for a few more metastable states. In particular the recently discovered[2] $Y_{1,5/2}{}^-(1765)$ clearly seems to belong to the bootstrap with $Y_{1,3/2}{}^-$ [assertion (c)]. An examination of Fig. 2 and the Chew-Frautschi plot[9,11] suggests that its mass should be greater than both $Y_{1,3/2}{}^-$ and $N_{1/2,5/2}{}^+$ (1660 and 1688 MeV, respectively) and smaller than $N_{3/2,7/2}{}^+$ at 1920 MeV—in agreement with experimental observation. Strictly speaking only a $(\pi,\Lambda)_2$ decay mode for $Y_1^*(1765)$ is allowed by our scheme, but in this high-energy range we should expect our crossing matrices to serve more as a qualitative guide than as a rigid selection rule for decay modes. Another resonance which can be accommodated by our approach is a $Y_{1,5/2}{}^+$ at still higher energy, since it belongs to an F state. This prediction can probably be identified with the Regge recurrence of the Σ member of the $J=\frac{1}{2}^+$ α octet in the notation of Glashow and Rosenfeld.[12] Two-body decays here will be $(\pi,\Lambda)_3$, $(\pi,\Sigma)_3$, and $(\bar{K},N)_3$.

We have considered here only the (N,Λ,Σ) baryon system. The Ξ particle has the same spin-parity and isospin as the nucleon and consequently should have the same family of resonances. That this is not so in practice serves to emphasize the limitations of our theory. Martin and Wali[13] have pointed out that coupled two-particle inelastic channels become important for the $\pi-\Xi$ system because of their proximity. Indeed the $\bar{K}\Sigma$ and $\bar{K}\Lambda$ thresholds are much closer to the $\pi-\Xi$ system than the corresponding $K\Sigma$ and $K\Lambda$ thresholds for the $\pi-N$ case.

In conclusion it is interesting to look at the baryon families from the point of view of self-consistency. We can consider the Λ particle as a P-wave bound state of $(\bar{K},N)_1$. K being pseudoscalar, Λ and N must thus have the same parity. Then again Σ is a $(\pi,\Lambda)_1$ system, hence (N,Λ,Σ) must all have the same parity. On the other hand, certain problems are raised as well. For all the bootstraps one should be able, in principle, to carry out calculations along the lines of Abers and Zemach[6] to obtain self-consistent masses for the two particles. But then again, many particles seem to be sustained by several bootstraps [for instance, $Y_1^*(1660)$ is sustained by $(\pi,\Sigma)_2$, $(\pi,\Lambda)_2$, and $(\bar{K}N)_2$ simultaneously]. Clearly a more refined theory is necessary which will treat all reactions on equal footing. It is to be hoped that a unique resonant mass will result from such a treatment. In particular, we wish to emphasize the importance of recent work[14] on establishing the interconnection between bootstraps and symmetries in strong interactions.

Note added in proof. While the introduction of SU(3) concepts is undoubtedly needed for a fuller treatment of the baryon resonances, such a program is, at present, beset with difficulties associated with nondegenerate masses. Hence, our approach, which takes advantage only of the lower SU(2) symmetry, should be useful, since it lends itself readily to calculations involving the masses. Furthermore, it will enable us to see more clearly, what features are indeed brought about by SU(3) that are not already contained in the lower symmetry.

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